

**TACTICAL GAMES  
&  
BEHAVIORAL SELF-ORGANIZATION**

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The interactive game theoretical approach to tactics and behavioral self-organization is developed. Though it uses the interactive game theoretical formalization of dialogues as psycholinguistic phenomena, the crucial role is played by the essentially new concept of a tactical game. Applications to the perception processes and related subjects (memory, recollection, image understanding, imagination) are discussed together with relations to the computer vision and pattern recognition (the dynamical formation of patterns and perception models during perception as a result of its self-organization) and computer games (modelling of the tactical behavior and self-organization, tactical RPG and elaboration of new tactical game techniques). The appendix is devoted to the operative computer games and the user programming of operative units in a multi-user online operative computer game.

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## Introduction

The mathematical formalism of interactive games, which extends one of ordinary games (see e.g.[1]) and is based on the concept of an interactive control, was recently proposed by the author [2] to take into account the complex composition of controls of a real human person, which are often complicated couplings of his/her cognitive and known controls with the unknown subconscious behavioral reactions. The goal of this article is to describe the tactical phenomena and behavioral self-organization in terms of interactive games. Though the crucial role is played by the interactive game theoretical concepts of dialogues and verbalizable games [3], the proposed constructions are essentially new as ideologically as technically. However, it should be specially emphasized that tactics is derived from the interactivity, and roughly speaking, the first may be thought as just an art to manipulate the Unknown, which manifests itself in the least, without making it known completely<sup>1</sup>. Otherwords, besides the reason tactics includes the sphere of “another effort of mind”, which is important not only in practice but also in theory<sup>2</sup>. And because this sphere is important for all moments of our life it is not strange that virtually all known forms of human activity such as scientific researches and economics, sport or military actions, fine or martial arts, literature, music and theatre, psychotherapy and even magic may be regarded as certain tactical games, which therefore have the universal meaning.

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<sup>1</sup> To my mind this is very reasonable if the ecological approach to the Unknown is adopted and we are not willing the least to be written by the golden letters of our Knowledge into the Red Book of the Universe.

<sup>2</sup> The simplest explanation why the transition from interactivity to tactics is necessary is that the description of interactive process is a tactical procedure (see e.g.[4]), otherwords, interactive games have their descriptors among tactical games whereas the class of tactical games contains all their descriptors and therefore is self-consistent. It is essential to mark that *the description of any interactive phenomena is interactive itself*.

It is well-known that the world around us is much deeper than what we see, are able to perceive or even to comprehend, so it is essential not only to understand the known forms of human (individual and collective) activity in terms of tactical games but also to try to unravel the abstract mathematical foundations of all tactical phenomena and thus to extend the sphere of tactical activity presumably into the abstract intelligible direction (certainly, completely conserving its practical orientation). Whereas the first purpose is the goal of the present article, the least will be discussed in the forthcoming one devoted to the game theoretical description of *dialectics*.

The article is organized in the following manner. Two first paragraphs are introductory. They are devoted to the general interactive games and the dialogues, respectively. The third paragraph is devoted to the new concept of tactical games. Interactive game theoretical models of behavioral self-organizations are discussed in the fourth paragraph. Other paragraphs contain special topics: in the fifth paragraph the attention is concentrated on tactics and behavioral self-organization constructed for the perception games and the kaleidoscope-roulettes in particular, the sixth one is devoted to the tactical interaction, tactical synthesis and tactical localization of control systems.

The article includes an appendix, where the operative computer games and the user programming of operative units in a multi-user online operative computer game are discussed.

The article is organized as series of definitions supplied by the clarifying remarks, which together describe the entire construction.

## I. INTERACTIVE GAMES

First of all, let us expose the basic principles of interactive games in general.

### 1.1. Interactive systems and games.

**Definition 1 [2].** An *interactive system* (with  $n$  *interactive controls*) is a control system with  $n$  independent controls coupled with unknown or incompletely known feedbacks (the feedbacks as well as their couplings with controls are of a so complicated nature that they can not be described completely). An *interactive game* is a game with interactive controls of each player.

Below we shall consider only deterministic and differential interactive systems. In this case the general interactive system may be written in the form:

$$(1) \quad \dot{\varphi} = \Phi(\varphi, u_1, u_2, \dots, u_n),$$

where  $\varphi$  characterizes the state of the system and  $u_i$  are the interactive controls:

$$u_i(t) = u_i(u_i^\circ(t), [\varphi(\tau)]|_{\tau \leq t}),$$

i.e. the independent controls  $u_i^\circ(t)$  coupled with the feedbacks on  $[\varphi(\tau)]|_{\tau \leq t}$ . One may suppose that the feedbacks are integrodifferential on  $t$ .

However, it is reasonable to consider the *differential interactive games*, whose feedbacks are purely differential. It means that

$$u_i(t) = u_i(u_i^\circ(t), \varphi(t), \dots, \varphi^{(k)}(t)).$$

A reduction of general interactive games to the differential ones via the introducing of the so-called *intention fields* was described in [2]. Below we shall consider the differential interactive games only if the opposite is not specified explicitly.

The interactive games introduced above may be generalized in the following ways.

The first way, which leads to the *indeterminate interactive games*, is based on the idea that the pure controls  $u_i^\circ(t)$  and the interactive controls  $u_i(t)$  should not be obligatory related in the considered way. More generally one should only postulate that there are some time-independent quantities  $F_\alpha(u_i(t), u_i^\circ(t), \varphi(t), \dots, \varphi^{(k)}(t))$  for the independent magnitudes  $u_i(t)$  and  $u_i^\circ(t)$ . Such claim is evidently weaker than one of Def.1. For instance, one may consider the inverse dependence of the pure and interactive controls:  $u_i^\circ(t) = u_i^\circ(u_i(t), \varphi(t), \dots, \varphi^{(k)}(t))$ .

The inverse dependence of the pure and interactive controls has a nice psychological interpretation. Instead of thinking of our action consisting of the conscious and unconscious parts and interpreting the least as unknown feedbacks which “dress” the first, one is able to consider our action as a single whole whereas the act of consciousness is in the extraction of a part which it declares as its “property”. So interpreted the function  $u_i^\circ(u_i, \varphi, \dots, \varphi^{(k)})$  realizes the “filtering” procedure.

The second way, which leads to the *coalition interactive games*, is based on the idea to consider the games with coalitions of actions and to claim that the interactive controls belong to such coalitions. In this case the evolution equations have the form

$$(2) \quad \dot{\varphi} = \Phi(\varphi, v_1, \dots, v_m),$$

where  $v_i$  is the interactive control of the  $i$ -th coalition. If the  $i$ -th coalition is defined by the subset  $I_i$  of all players then

$$v_i = v_i(\varphi(t), \dots, \varphi^{(k)}(t), u_j^\circ | j \in I_i).$$

Certainly, the intersections of different sets  $I_i$  may be non-empty so that any player may belong to several coalitions of actions. Def.1 gives the particular case when  $I_i = \{i\}$ .

The coalition interactive games may be an effective tool for an analysis of the collective decision making in the real coalition games that spread the applicability of the elaborating interactive game theory to the diverse problems of sociology.

*Remark 1.* One is able to consider interactive games of discrete time in the similar manner.

*Remark 2.* If one suspect that the explicit dependence of the feedbacks on the derivatives of  $\varphi$  is not correct because they are determined via the evolution equations governed by the interactive controls, it is reasonable to use the inverse dependence of pure and interactive controls.

**1.2. The  $\varepsilon$ -representations.** Interactive games are games with incomplete information by their nature. However, this incompleteness is in the unknown feedbacks, not in the unknown states. The least situation is quite familiar to specialists in game theory and there is a lot of methods to have deal with it. For instance, the unknown states are interpreted as independent controls of the virtual players and some assumptions on their strategies are done. To transform interactive games into the games with an incomplete information on the states one can use the following trick, which is called the  $\varepsilon$ -representation of the interactive game.

**Definition 2.** The  $\varepsilon$ -representation of the differential interactive game is a representation of the interactive controls  $u_i(t)$  in the form

$$u_i(t) = u_i(u_i^\circ(t), \varphi(t), \dots, \varphi^{(k)}(t); \varepsilon_i(t))$$

with the *known* function  $u_i$  of its arguments  $u_i^\circ, \varphi, \dots, \varphi^{(k)}$  and  $\varepsilon_i$ , whereas

$$\varepsilon_i(t) = \varepsilon_i(u_i^\circ(t), \varphi(t), \dots, \varphi^{(k)}(t))$$

is the *unknown* function of  $u_i^\circ$  and  $\varphi, \dots, \varphi^{(k)}$ .

*Remark 3.* The derivatives of  $\varphi$  may be excluded from the feedbacks in the way described above.

*Remark 4.* One is able to consider the  $\varepsilon$ -representations of the indeterminate and coalition interactive games.

$\varepsilon_i$  are interpreted as parameters of feedbacks and, thus, characterize the internal existential states of players. It motivates the notation  $\varepsilon$ . Certainly,  $\varepsilon$ -parameters are not really states being the unknown functions of the states and pure controls, however, one may sometimes to apply the standard procedures of the theory of games with incomplete information on the states. For instance, it is possible to regard  $\varepsilon_i$  as controls of the virtual players. The naïvely introduced virtual players only double the ensemble of the real ones in the interactive games but in the coalition interactive games the collective virtual players are observed. More sophisticated procedures generate ensembles of virtual players of diverse structure.

Precisely, if the derivatives of  $\varphi$  are excluded from the feedbacks (at least, from the interactive controls  $u_i$  as functions of the pure controls, states and the  $\varepsilon$ -parameters) the evolution equation will have the form

$$(3) \quad \dot{\varphi}(t) = \Phi(\varphi, u_1(u_1^\circ(t), \varphi(t); \varepsilon_1(t)), \dots, u_n(u_n^\circ(t), \varphi(t); \varepsilon_n(t))),$$

so it is consistent to regard the equations as ones of the controlled system with the ordinary controls  $u_1, \dots, u_n, \varepsilon_1, \dots, \varepsilon_n$ . One may consider a new game postulating that these controls are independent. Such game will be called the ordinary differential game associated with the  $\varepsilon$ -representation of the interactive game.

**1.3. Hidden interactivity of ordinary differential games [5].** Let us consider an arbitrary ordinary differential game with the evolution equations

$$\dot{\varphi} = \Phi(\varphi, u_1, u_2, \dots, u_n),$$

where  $\varphi$  characterizes the state of the system and  $u_i$  are the ordinary controls. Let us fix a player. For simplicity of notations we shall suppose that it is the first one. As a rule the players have their algorithms of predictions of the behaviour of other players. For a fixed moment  $t_0$  of time let us consider the prediction of the first player for the game. It consists of the predicted controls  $u_{[t_0];i}^\circ(t)$  ( $t > t_0$ ;  $i \geq 2$ ) of all players and the predicted evolution of the system  $\varphi_{[t_0]}^\circ(t)$ . Let us fix  $\Delta t$  and consider the real and predicted controls for the moment  $t_0 + \Delta t$ . Of course, they may be different because other players use another algorithms for the game prediction. One may interpret the real controls  $u_i(t)$  ( $t = t_0 + \Delta t$ ;  $i \geq 2$ ) of other

players as interactive ones whereas the predicted controls  $u_{[t_0];i}^\circ(t)$  as pure ones, i.e. to postulate their relation in the form:

$$u_i(t) = u_i(u_{[t-\Delta t];i}^\circ(t); \varphi_{[t_0]}^\circ(\tau) | \tau \leq t).$$

In particular, the feedbacks may be either reduced to differential form via the introducing of the intention fields or simply postulated to be differential. Thus, we constructed an interactive game from the initial ordinary game. One may use  $\varphi(\tau)$  as well as  $\varphi_{[t_0]}^\circ(\tau)$  in the feedbacks.

Note that the controls of the first player may be also considered as interactive if the corrections to the predictions are taken into account when controls are chosen.

The obtained construction may be used in practice to make more adequate predictions. Namely, *a posteriori* analysis of the differential interactive games allows to make the short-term predictions in such games [6]. One should use such predictions instead of the initial ones. Note that at the moment  $t_0$  the first player knows the pure controls of other players at the interval  $[t_0, t_0 + \Delta t]$  whereas their real freedom is interpreted as an interactivity of their controls  $u_i(t)$ . So it is reasonable to choose  $\Delta t$  not greater than the admissible time depth of the short-term predictions. Estimations for this depth were proposed in [6].

Naïvely, the proposed idea to improve the predictions is to consider deviations of the real behaviour of players from the predicted ones as a result of the interactivity, then to make the short-term predictions taking the interactivity into account and, thus, to receive the corrections to the initial predictions. Such corrections may be regarded as “psychological” though really they are a result of different methods of predictions used by players.

*Remark 5.* The interpretation of the ordinary differential game as an interactive game also allows to perform the strategical analysis of interactive games. Indeed, let us consider an arbitrary differential interactive game  $A$ . Specifying its  $\varepsilon$ -representation one is able to construct the associated ordinary differential game  $B$  with the doubled number of players. Making some predictions in the game  $B$  one transform it back into an interactive game  $C$ . Combination of the strategical long-term predictions in the game  $B$  with the short-term predictions in  $C$  is often sufficient to obtain the adequate strategical prognosis for  $A$ .

*Remark 6.* The interpretation of the ordinary differential game as an interactive game is especially useful in situations, when the goals of players are not known precisely to each other and some more or less rough suppositions are made.

## II. DIALOGUES AND VERBALIZABLE GAMES

**2.1. Dialogues.** Let us now expose the interactive game formalism for a description of dialogues as psycholinguistic phenomena [3]. First of all, note that one is able to consider interactive games of discrete time as well as interactive games of continuous time above.

**Defintion 3A (the naïve definition of dialogues) [3].** The *dialogue* is a 2-person interactive game of discrete time with intention fields of continuous time.

The states and the controls of a dialogue correspond to the speech whereas the intention fields describe the understanding.

Let us give the formal mathematical definition of dialogues now.

**Definition 3B (the formal definition of dialogues) [3].** The *dialogue* is a 2-person interactive game of discrete time of the form

$$(4) \quad \varphi_n = \Phi(\varphi_{n-1}, \vec{v}_n, \xi(\tau) | t_{n-1} \leq \tau \leq t_n).$$

Here  $\varphi_n = \varphi(t_n)$  are the states of the system at the moments  $t_n$  ( $t_0 < t_1 < t_2 < \dots < t_n < \dots$ ),  $\vec{v}_n = \vec{v}(t_n) = (v_1(t_n), v_2(t_n))$  are the interactive controls at the same moments;  $\xi(\tau)$  are the intention fields of continuous time with evolution equations

$$(5) \quad \dot{\xi}(t) = \Xi(\xi(t), \vec{u}(t)),$$

where  $\vec{u}(t) = (u_1(t), u_2(t))$  are continuous interactive controls with  $\varepsilon$ -represented couplings of feedbacks:

$$u_i(t) = u_i(u_i^\circ(t), \xi(t); \varepsilon_i(t)).$$

The states  $\varphi_n$  and the interactive controls  $\vec{v}_n$  are certain known functions of the form

$$(6) \quad \begin{aligned} \varphi_n &= \varphi_n(\vec{\varepsilon}(\tau), \xi(\tau) | t_{n-1} \leq \tau \leq t_n), \\ \vec{v}_n &= \vec{v}_n(\vec{u}^\circ(\tau), \xi(\tau) | t_{n-1} \leq \tau \leq t_n). \end{aligned}$$

Note that the most nontrivial part of mathematical formalization of dialogues is the claim that the states of the dialogue (which describe a speech) are certain “mean values” of the  $\varepsilon$ -parameters of the intention fields (which describe the understanding).

*Important.* The definition of dialogue may be generalized on arbitrary number of players and below we shall consider any number  $n$  of them, e.g.  $n = 1$  or  $n = 3$ , though it slightly contradicts to the common meaning of the word “dialogue”.

**2.2. The verbalization of interactive games.** An embedding of dialogues into the interactive game theoretical picture generates the reciprocal problem: how to interpret an arbitrary differential interactive game as a dialogue. Such interpretation will be called the *verbalization*.

**Definition 4 [3].** A differential interactive game of the form

$$\dot{\varphi}(t) = \Phi(\varphi(t), \vec{u}(t))$$

with  $\varepsilon$ -represented couplings of feedbacks

$$u_i(t) = u_i(u_i^\circ(t), \varphi(t), \dot{\varphi}(t), \ddot{\varphi}(t), \dots, \varphi^{(k)}(t); \varepsilon_i(t))$$

is called *verbalizable* if there exist a *posteriori* partition  $t_0 < t_1 < t_2 < \dots < t_n < \dots$  and the integrodifferential functionals

$$(7) \quad \begin{aligned} \omega_n(\vec{\varepsilon}(\tau), \varphi(\tau) | t_{n-1} \leq \tau \leq t_n), \\ \vec{v}_n(\vec{u}^\circ(\tau), \varphi(\tau) | t_{n-1} \leq \tau \leq t_n) \end{aligned}$$

such that

$$(8) \quad \omega_n = \Omega(\omega_{n-1}, v_n; \varphi(\tau) | t_{n-1} \leq \tau \leq t_n).$$

The verbalizable differential interactive games realize a dialogue in sense of Def.3.

*Remark 7.* One may include  $\omega_n$  explicitly into the evolution equations for  $\varphi$

$$\dot{\varphi}(\tau) = \Phi(\varphi(\tau), \vec{u}(\tau); \omega_n), \quad \tau \in [t_n, t_{n+1}].$$

as well as into the feedbacks and their couplings.

The main heuristic hypothesis is that all differential interactive games “which appear in practice” are verbalizable. The verbalization means that the states of a differential interactive game are interpreted as intention fields of a hidden dialogue and the problem is to describe such dialogue completely. If a differential interactive game is verbalizable one is able to consider many linguistic (e.g. the formal grammar of a related hidden dialogue) or psycholinguistic (e.g. the dynamical correlation of various implications) aspects of it.

During the verbalization it is a problem to determine the moments  $t_i$ . A way to the solution lies in the structure of  $\varepsilon$ -representation. Let the space  $E$  of all admissible values of  $\varepsilon$ -parameters be a CW-complex. Then  $t_i$  are just the moments of transition of the  $\varepsilon$ -parameters to a new cell.

**2.3. Psycholinguistic encoding/decoding of dialogues.** Let's now describe one practically valuable procedure. If one has a dialogue it is possible to consider it as a verbalizable game and, hence, as an interactive game. Then one is able to forget the initial dialogue structure of this game and to make an attempt to verbalize it. Certainly, a different dialogue may be obtained in such way. This procedure will be called the *psycholinguistic encoding* of the initial dialogue, whereas the reciprocal one will be called the *psycholinguistic decoding*.

In practice, the psycholinguistic encoding/decoding may include the change of the communication medium, e.g. the phonic-verbal dialogue may be encoded as visual-figurative one (the *illustration*) and vice versa (the *ecphrasis*). The analysis of psycholinguistic encoding/decoding may be useful for a clarification of nature of the speech-script dualism of language as well as for an understanding of linguistic aspects of various forms of art such as dance, martial arts, etc.

### III. TACTICAL GAMES

Tactics as it will be defined below is derived from two independent concepts: the parametric interactive games and external controls on one hand and the comments to dialogues on another hand.

**3.1. Parametric interactive games and external controls.** An interactive game may depend on the additional parameters. Such dependence is of two forms. First, parameters may appear in the evolution equations:

$$(9A) \quad \dot{\varphi} = \Phi(\varphi, u_1, u_2, \dots, u_n; \lambda).$$

Here,  $\lambda$  is a collective notation for parameters. Second, parameters may appear in feedbacks:

$$(9B) \quad u_i(t) = u_i(u_i^\circ(t), \varphi(t), \dots, \varphi^{(k)}(t); \lambda).$$

The dependence of  $u_i$  on  $\lambda$  is either unknown (incompletely known) or known. The least means that  $\partial u_i / \partial \lambda$  may be expressed via  $u_i$  as a function of other variables (such expression are integrodifferential on these variables). Both variants of parametric dependence of interactive game may be combined together.

The additional parameters may realize the external controls. In this situation they depend on time:

$$\lambda = \lambda(t).$$

In practice, such situation appear in the teaching systems. The parameters are interpreted as controls of a teacher. This example is rather typical. It shows that the controls  $\lambda(t)$  may be considered as “slow” whereas the interactive controls  $u_i(t)$  as “quick”.

Of course, one is able to introduce the slow controls  $\lambda(t)$ , which belong to the same players as the interactive controls  $u_i(t)$  or to their coalitions. And, certainly, the slow controls of discrete time may be considered. One may suspect that the discrete time controls  $\lambda_n$  realize a convenient approximation for the slow controls  $\lambda(t)$ , which is timer in practice.

The slow controls may be interactive.

If dependence of  $u_i$  on  $\lambda$  is known and one consider the  $\varepsilon$ -representation of feedbacks it is either postulated that  $\varepsilon$ -parameters do not depend on  $\lambda$  or claimed that  $\partial\varepsilon/\partial\lambda$  is expressed via  $\varepsilon$  as a function of other arguments.

**3.2. Dialogue and comments.** Let us consider a  $n$ -person dialogue

$$\omega_n = \Omega(\omega_{n-1}, \vec{v}_n, \xi(\tau) | t_{n-1} \leq \tau \leq t_n)$$

with the discrete time interactive controls  $\vec{v}_n$  and the intention fields governed by the evolution equations

$$\dot{\xi}(t) = \Xi(\xi(t), \vec{u}(t)),$$

where  $\vec{u}(t)$  are the continuous interactive controls with  $\varepsilon$ -represented couplings of feedbacks:

$$u_i(t) = u_i(u_i^\circ(t), \xi(t); \varepsilon_i(t)).$$

The states  $\varphi_n$  and the interactive controls  $\vec{v}_n$  are expressed as

$$\begin{aligned} \omega_n &= \omega_n(\vec{\varepsilon}(\tau), \xi(\tau) | t_{n-1} \leq \tau \leq t_n), \\ \vec{v}_n &= \vec{v}_n(\vec{u}^\circ(\tau), \xi(\tau) | t_{n-1} \leq \tau \leq t_n). \end{aligned}$$

The discrete time *comments*  $\vartheta_n$  to the dialogue are defined recurrently as

$$(10) \quad \vartheta_n = \Theta(\vartheta_{n-1}, \omega_n, v_n).$$

Comments to the dialogue at the fixed moment  $t_n$  contain various information on the dialogue. For instance, one may to raise a problem to restore some features of a dialogue from certain comments or alternatively what features of a dialogue may be restored from the fixed comment.

The main difference of the comments  $\vartheta_n$  from the states  $\omega_n$  is the absence of expressions of the first via  $\vec{\varepsilon}(\tau)$  and  $\xi(\tau)$  ( $t_{n-1} \leq \tau \leq t_n$ ).

Comments are applied to the verbalizable games in the same way.

**3.3. Tactical games.** Tactical games combine both mechanisms defined above.

**Definition 5.** The *tactical game* is a parametric verbalizable game with comments, in which the parameters are of discrete time and coincide with the comments.

It is really wonderful that such simple definition is applicable to a very huge class of phenomena. However, it is so! As it was marked above virtually all known forms of human activity such as scientific researches and economics, sport or military actions, fine or martial arts, literature, music and theatre, psychotherapy and even magic may be regarded as certain tactical games. Trying to improve the model I have no found any wider concept, whose using is necessary and effective, whereas the notion of tactical game may describe these phenomena very correctly.

*Remark 8.* The pairs  $(v_n, \vartheta_n)$  of discrete time interactive controls and the comments will be called the *tactical actions*, whereas the continuous time interactive controls  $u_i(t)$  will be called the *instant actions*. The tactical actions may be involved as in the evolution equations as in the interactivity.

*Remark 9.* The description(-construction) of interactive games in sense of [4] is a tactical procedure.

**3.4. Applications to computer games (tactical action games and the modelling of tactical behavior).** Though some computer games claim that they are tactical it is not so. Really tactics is not involved in the rules of such games and is accidental. Otherwords, a player may perform the tactical actions but they are not obligatory. The real actions are instant only and their tactical interpretation is not substantial. Moreover, artificial players do not perform any really tactical actions except the actions following a behavioral pattern. The precise definition of a tactical game allows to create, first, the tactical action games, where tactical actions are substantial, second, to perform the computer modelling of tactical behavior for the artificial players. It is not difficult and is very interesting to do. In present, the author effectively uses the tactical game constructions in elaboration of tactical computer games. However, the discussion of practical questions is a bit beyond the aim of this article.

#### IV. BEHAVIORAL SELF-ORGANIZATION

This paragraph is devoted to the tactical game modelling of the behavioral self-organization.

**4.1. Tactical behavioral self-organization.** The tactical properties of players may change during the game. Note that the tactics is defined by the dependence of the evolution equations and feedbacks on the comments and by the function  $\Theta$ , which determines the comments recurrently. One may fix the dependence on the comments and vary the function  $\Theta$ . The procedures of improvement of  $\Theta$ , which goal is an adaptation of a player, form the *tactical behavioral self-organization*. Thus, the tactical behavioral self-organization is a form of functional self-organization. Numerous methods of modelling of such self-organization may be, therefore, adopted.

Tactical behavioral self-organization should have a lot of real applications in all forms of human activity, where tactics appears. There are several directions of such applications: (1) the teaching, (2) the human adaptation and self-regulation systems, (3) semi-artificial human-computer systems, (4) purely artificial computer systems. The most of them should be based on the results, which will be obtained in the computer games.

**4.2. Applications to computer games (artificial intelligence and tactical self-organization, tactical RPG).** Combinations of the artificial intelligence and tactical self-organization are applicable to computer games. In this way the truly tactical RPG may be created. Note that virtually all known RPG are not tactical and therefore do not use the tactical behavioral self-organization (in fact, they use any behavioral self-organization only episodically, the most elaborated forms of self-organization are applied by the Japanese but they are not widely distributed). The author, who actively works in this direction, suspects that it has a lot of perspectives.

## V. TACTICS AND BEHAVIORAL SELF-ORGANIZATION FOR PERCEPTION GAMES

This paragraph is devoted to applications of tactical games to the perception processes and related subjects (memory, recollection, image understanding, imagination) as well as to the computer vision and pattern recognition (the dynamical formation of patterns and perception models during perception as a result of its self-organization).

**5.1. Perception games and kaleidoscope-roulettes [7].** Perception processes was understood in the interactive game theoretical terms in the articles [7]. The main concept is one of the perception game, which is exposed below.

**Definition 6.** The *perception game* is a multistage verbalizable game (no matter finite or infinite) for which the intervals  $[t_i, t_{i+1}]$  are just the sets. The conditions of their finishing depends only on the current value of  $\varphi$  and the state of  $\omega$  at the beginning of the set. The initial position of the set is the final position of the preceeding one.

Practically, the definition describes the discrete character of the perception and the image understanding. For example, the goal of a concrete set may be to perceive or to understand certain detail of the whole image. Another example is a continuous perception of the moving or changing object.

Note that the definition of perception games is applicable to various forms of perception, though the most interesting one is the visual perception. The proposed definition allows to take into account the dialogical character of the image understanding and to consider the visual perception, image understanding and the verbal (and nonverbal) dialogues together. It may be extremely useful for the analysis of collective perception, understanding and controlling processes in the dynamical environments – sports, dancings, martial arts, the collective controlling of moving objects, etc. On the other hand this definition explicates the self-organizing features of human perception, which may be unraveled by the game theoretical analysis. And, finally, the definition put a basis for a systematical application of the linguistic (e.g. formal grammars) and psycholinguistic methods to the image understanding as a verbalizable interactive game with a mathematical rigor.

Let's now consider an interesting class of perception games, the kaleidoscope-roulettes.

The kaleidoscope-roulette is a result of the attempt to combine the kaleidoscope, one of the simplest and effective visual game, with the roulette essentially using the elements of randomness and the treatment of resonances. The main idea is to substitute random sequences of roulette by the quasirandom sequences, which

may be generated by the interactive kaleidoscope. The obtained formal definition is below.

**Definition 7.** *Kaleidoscope-roulette* is a perception game with a quasirandom sequence of quantites  $\{\omega_n\}$ .

Certainly, the explicit form of functionals (7) is not known to the players.

Many concrete versions of kaleidoscope-roulettes are constructed. Though they are naturally associated with an entertainment their real applications may be far beyond it due to their origin and the abstract character of their definition.

Kaleidoscope-roulettes are very interesting due to the resonance phenomena, which may appear in them. Indeed, though the sequence  $\{\omega_n\}$  is quasirandom the equations (8) for them may have the resonance solutions. The resonance means a dynamical correlation of two quasirandom sequences  $\{v_n\}$  and  $\{\omega_n\}$  whatever  $\varphi$  is realized. In such case the quantities  $\{v_n\}$  may be comprehended as “fortune”, what is not senseless in contrast to the ordinary roulette. However,  $v_n$  are *interactive* controls and their explicit dependence on  $\vec{u}^o$  and  $\varphi$  is not known. Nevertheless, one is able to use a *posteriori* analysis and short-term predictions based on it (cf.[6]) if the time interval  $\Delta t$  in the short-term predictions is not less than the interval  $t_{n+1} - t_n$ . To do it one should slightly improve constructions of [6] to take the discrete-time character of  $v_n$  into account. It allows to perform the short-term controlling of the resonances in a kaleidoscope-roulette if they are observed. The conditions of applicability of short-time predictions to the controlling of resonances may be expressed in the following form: one should claim that *variations of the interactivity should be slower than the change of sets in the considered multistage game*.

*Remark 10.* The possibility to control resonances by  $v_n$  using its short-term predictions does not contradict to its quasirandomness, because  $v_n$  is quasirandom with respect to  $v_{n-1}$  but not to  $\varphi(\tau)$  ( $\tau \in [t_n, t_{n+1}]$ ).

**5.2. Tactics and behavioral self-organization for perception games. Perceptive oracles.** Many “representative” mechanisms in perception processes have the tactical origin. One should include the memory, the recollection, the image understanding and the imagination. These phenomena may be described in terms of the *tactical perception games*. Procedures of tactical behavioral self-organization may be applied to the memory strengthening, the intensification of creativity during imagination and other psychological problems.

Below we shall discuss an interesting example related to the kaleidoscope-roulettes, the perceptive oracle.

**Definition 8.** The parametric kaleidoscope-roulette, which is a tactical game, is called a *perceptive oracle*.

Thus, the perceptive oracle is a kaleidoscope-roulette if the comments are frozen and in this case the sequence  $\{\omega_n\}$  is quasirandom. However, it may be not so if the comments form tactical actions. In the resonances the sequence  $\{\omega_n\}$  may have some laws. The weaker forms of relations may also appear, e.g. the integrals  $K_\alpha(\omega_n, \vartheta_n, \omega_{n-1}, \vartheta_{n-1}, \dots, \omega_{n-k}, \vartheta_{n-k}) \equiv k_\alpha$  may exist for several subsequent  $n$ . This explain the choice of the name ”perceptive oracle”. The appearing of resonances is manifested by the omens [5:Rem.7]. So such omens allow to perform some predictions in the perceptive oracle.

*Remark 11.* The perceptive oracle may be regarded as a new tactical game technique and, thus, may be used for the generation of other tactical game techniques for the computer games. For instance, it will be very interesting to combine perceptive oracle with the conveniently generalized domino technique.

Some words should be said on other applications of the tactical perception game formalism. Besides the phenomena of human perception such games may be used for elaboration of man-made systems. Thus, the tactical perception games are a natural framework for some problems of the computer vision and pattern recognition, e.g. the dynamical formation of patterns and perception models during perception as a result of its self-organization.

## VI. TACTICAL INTERACTION, TACTICAL SYNTHESIS AND TACTICAL LOCALIZATION OF CONTROL SYSTEMS

Note that in a tactical game the comments form a control system with the initially fixed control algorithm (and tactical behavioral self-organization is interpreted as a self-developing of this algorithm). Indeed, if one consider the control system as unity of structural and functional aspects, the generation of comments is a structural aspect whereas their representation as parameters of evolution or interactivity forms a functional aspect. Application of the tactical game formalism to the simultaneous functioning of several control systems (their interaction, synthesis and localization) is discussed below.

**6.1. Tactical interaction of control systems.** Let us consider two control systems represented as tactical games defined by the evolution equations

$$\dot{\varphi}_1 = \Phi_1(\varphi_1, \vec{u}_1; \vartheta_1)$$

and

$$\dot{\varphi}_2 = \Phi_2(\varphi_2, \vec{u}_2; \vartheta_2)$$

with  $\varepsilon$ -represented couplings of feedbacks

$$u_{1,i} = u_{1,i}(u_{1,i}^o, \varphi_1, \dot{\varphi}_1, \ddot{\varphi}_1, \dots, \varphi_1^{(k)}; \varepsilon_{1,i}, \vartheta_1)$$

and

$$u_{2,i} = u_{2,i}(u_{2,i}^o, \varphi_2, \dot{\varphi}_2, \ddot{\varphi}_2, \dots, \varphi_2^{(k)}; \varepsilon_{2,i}, \vartheta_2).$$

The integrodifferential functionals (7) have the form

$$\begin{aligned} \omega_{j,n}(\vec{\varepsilon}_j(\tau), \varphi_j(\tau) | t_{n-1} \leq \tau \leq t_n), \\ \vec{v}_{j,n}(\vec{u}_j^o(\tau), \varphi_j(\tau) | t_{n-1} \leq \tau \leq t_n) \end{aligned}$$

and the relations (8)

$$\omega_{j,n} = \Omega_j(\omega_{j,n-1}, v_{j,n}; \varphi_j(\tau) | t_{n-1} \leq \tau \leq t_n)$$

hold ( $j = 1, 2$ ). The comments  $\vartheta_1$  and  $\vartheta_2$  are defined recurrently as

$$\vartheta_{1,n} = \Theta_1(\vartheta_{1,n-1}, \omega_{1,n}, v_{1,n})$$

and

$$\vartheta_{2,n} = \Theta_2(\vartheta_{2,n-1}, \omega_{2,n}, v_{2,n}).$$

The *tactical interaction* is realized by the addition of the interaction terms into the recurrent formulas for  $\vartheta_j$  to produce the interdetermination of comments:

$$\vartheta_{1,n} = \Theta_1(\vartheta_{1,n-1}, \omega_{1,n}, v_{1,n}) + \tilde{\Theta}_{1,2}^{\text{int}}(\vartheta_{1,n-1}, \vartheta_{2,n-1}, \omega_{1,n}, v_{1,n})$$

and

$$\vartheta_{2,n} = \Theta_2(\vartheta_{2,n-1}, \omega_{2,n}, v_{2,n}) + \tilde{\Theta}_{2,1}^{\text{int}}(\vartheta_{2,n-1}, \vartheta_{1,n-1}, \omega_{2,n}, v_{2,n}).$$

**6.2. Tactical synthesis of control systems.** Let us consider  $N$  control systems represented as tactical games defined by the evolution equations

$$\dot{\varphi}_j = \Phi_j(\varphi_j, \vec{u}_j; \vartheta_j)$$

( $j = 1, 2, \dots, N$ ) with  $\varepsilon$ -represented couplings of feedbacks

$$u_{j,i} = u_{j,i}(u_{j,i}^{\circ}, \varphi_j, \dot{\varphi}_j, \ddot{\varphi}_j, \dots, \varphi_j^{(k)}; \varepsilon_{j,i}, \vartheta_j).$$

The integrodifferential functionals (7) have the form

$$\begin{aligned} \omega_{j,n}(\vec{\varepsilon}_j(\tau), \varphi_j(\tau) | t_{n-1} \leq \tau \leq t_n), \\ \vec{v}_{j,n}(\vec{u}_j^{\circ}(\tau), \varphi_j(\tau) | t_{n-1} \leq \tau \leq t_n) \end{aligned}$$

and the relations (8)

$$\omega_{j,n} = \Omega_j(\omega_{j,n-1}, v_{j,n}; \varphi_j(\tau) | t_{n-1} \leq \tau \leq t_n)$$

hold. The comments  $\vartheta_j$  are defined recurrently as

$$\vartheta_{j,n} = \Theta_j(\vartheta_{j,n-1}, \omega_{j,n}, v_{j,n}).$$

The *tactical synthesis* is realized by the redefinition of the recurrent formulas for  $\vartheta_j$  to produce the unification of comments:

$$\vartheta_{j,n} = \tilde{\Theta}_j(\vartheta_{1,n-1}, \dots, \vartheta_{N,n-1}, \omega_{1,n}, \dots, \omega_{N,n}, v_{1,n}, \dots, v_{N,n}).$$

The functions  $\tilde{\Theta} = (\tilde{\Theta}_1, \dots, \tilde{\Theta}_N)$  determines the synthesis. It may has various internal structure, which is characterized by the set of real arguments of functions  $\tilde{\Theta}_j$  and their hierarchical structure. It presupposes that  $\tilde{\Theta}_j$  depend not on all triples  $(\vartheta_j, \omega_j, v_j)$  and various triples may appear in  $\tilde{\Theta}_j$  in coalitions of different form and nature. One may think that the functions  $\tilde{\Theta} = (\tilde{\Theta}_1, \dots, \tilde{\Theta}_N)$  are constructed from the functions  $\Theta = (\Theta_1, \dots, \Theta_N)$  using some operations, which realize the synthesis. Thus, the synthesis may be performed in several steps and be described by an algorithm. Optimization problems for the synthesis and its construction naturally arise.

**6.3. Tactical localization of control systems.** Tactical localization is a procedure reciprocal to the tactical synthesis. Let us consider a control system represented as a tactical game

$$\dot{\varphi} = \Phi(\varphi, \vec{u}; \vartheta)$$

with  $\varepsilon$ -represented couplings of feedbacks

$$u_i = u_i(u_i^\circ, \varphi, \dot{\varphi}, \ddot{\varphi}, \dots, \varphi^{(k)}; \varepsilon_i, \vartheta).$$

The integrodifferential functionals (7) have the form

$$\begin{aligned}\omega_n(\vec{\varepsilon}(\tau), \varphi(\tau) | t_{n-1} \leq \tau \leq t_n), \\ \vec{v}_n(\vec{u}^\circ(\tau), \varphi(\tau) | t_{n-1} \leq \tau \leq t_n)\end{aligned}$$

and the relations (8)

$$\omega_n = \Omega(\omega_{n-1}, v_n; \varphi(\tau) | t_{n-1} \leq \tau \leq t_n)$$

hold. The comments  $\vartheta$  are defined recurrently as

$$\vartheta_n = \Theta(\vartheta_{n-1}, \omega_n, v_n).$$

To perform the *tactical localization* means to represent the control system as a result of a tactical synthesis of  $N$  partial control systems.

The localization of control systems is often used to minimize the resource expenses and to reduce the information circuits. The tactical localization may be useful in this way.

## APPENDIX A. OPERATIVE COMPUTER GAMES

The appendix is devoted to the operative computer games and the user programming of operative units in a multi-user online operative computer game.

**A.1. Tactics and operative art. Operative computer games.** The concept of the operative game has its origin in the Russian tradition of military science, where operative art is concerned as the intermediate component of military art between tactics and strategy. Structurally, the difference between tactics and operative art is that tactics investigates the controlling of the interactively controlled systems as it was described in the main text of this article, whereas operative art has deal with the controlling of such systems, which are simultaneously the control ones (and their controls may be also interactive). Therefore, the operative units are in fact the complex tactical formations.

The most natural models for the operative art are the operative computer games (OCG), when computer is modelling the behavior of various operative units and is functioning as the control system. The player sends interactively the commands to the operative units, which are performed by the computer. However, a way of their realization is not known completely to the player. For instance, the distribution of functions between tactical units as well as tactical features of these units may be unknown incompletely.

Note that tactics has deal with tactical units whereas the operative art has deal with operative units, which are tactical formations.

Usually, operative units are represented as schematic non-figurative signs for the tactical formations. Thus, the game field is realized in two different separate views: one has, first, the observable field of figurative tactical units and, second, the schematic field of the corresponding operative units on the screen of the monitor.

**A.2. Programming of operative units in operative computer games.** It is an interesting problem of the user programming of operative units in a multi-user online operative computer game. Ordinarily, such game is a multi-stage one, so the programming of operative units is done between the sets. There are two variants. First, the game does not provide the players by any sources for such programming. In this case an individual player should write his/her own algorithms for operative units and then to adapt them to the game. Second, the game may include the unified sources for the operative unit programming. For instance, it can specify the programming language and special libraries. Certainly, the language should be simple, close to the ordinary programming languages, compatible with them and effective. Undoubtedly, none of elaborated programming languages is not specially adapted for the problem, however, separate features are useful. The compatibility essentially restricts the effectiveness because known languages were not created for the multi-user online games. However, it will be a problem of the whole computer game community if somebody decides to include the programming of operative units in the proposed game. I may only hope that the game “for the programmers” will be interesting enough to compensate such social difficulties and will stimulate a collective activity on the boundary of computer games and the programming art.

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